Volatility Managed Portfolios

Alan Moreira Tyler Muir

Yale University

Cornell University
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What do we do?

1. Volatility managed portfolios: scale aggregate priced factor by $1/\sigma_t^2$

2. Motivation: risky asset demand

$$w_t = \frac{1}{\gamma} \frac{E_t(R_{t+1})}{Var_t(R_{t+1})}$$

3. Volatility doesn't forecast returns $\Rightarrow$ volatility timing beneficial

What do we find?

Volatility managed portfolios

1. increase Sharpe ratios, generate large alpha on original factors
2. take *less* risk in recessions when σ high
3. *sells* after market crashes (1929, 1987, 2008), selling typically viewed as mistake

Outline

1. Vol managed portfolios empirically
2. Who should volatility time?
    - Large utility benefits for mean variance investor
    - Long horizon investors
3. General equilibrium
    - Price of risk negatively related to vol, contrary to standard theories

Data

- Factors: Market, SMB, HML, Momentum, Profitability, ROE, Investment, Carry (FX)
- Daily and monthly data for each factor
- Sample: 1926-2015 (Mkt, SMB, HML, Momentum), Post 1960 for the rest
- Also include unconditional mean-variance efficient portfolio (MVE) from factors
- All numbers annualized

Managed volatility factors

1. Let f_{t+1} be an excess return, construct

$$f_{t+1}^{\sigma} = \frac{c}{\sigma_t^2} \times f_{t+1}$$

   - σ_t previous month realized volatility
   - choose c so f^{σ} has same unconditional variance as f

2. Regression:

$$f_{t+1}^{\sigma} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$$

   - Formally show: α large when vol doesn't forecast returns

Volatility managed factors: alphas

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}	(7) MVE^{σ}	(8) FX^{σ}	(9) ROE^{σ}	(10) IA^{σ}
MktRF	0.61									
	(0.05)									
SMB		0.62								
		(0.08)								
HML			0.57							
			(0.07)							
Mom				0.47						
				(0.07)						
RMW					0.62					
					(0.08)					
CMA						0.68				
						(0.05)				
MVE							0.58			
							(0.03)			
Carry								0.71		
								(0.08)		
ROE									0.63	
									(0.07)	
IA										0.68
										(0.05)
α	4.86	-0.58	1.97	12.51	2.44	0.38	4.12	2.78	5.48	1.55
	(1.56)	(0.91)	(1.02)	(1.71)	(0.83)	(0.67)	(0.77)	(1.49)	(0.97)	(0.67)
N	1,065	1,065	1,065	1,060	621	621	1,060	360	575	575
R2	0.37	0.38	0.32	0.22	0.38	0.46	0.33	0.51	0.40	0.47
rmse	51.39	30.44	34.92	50.37	20.16	17.55	25.34	21.78	23.69	16.58

Volatility managed factors

How much do we increase Sharpe ratio / expand MVE frontier?

$$\sqrt{SR_{old}^2 + \left(\frac{\alpha}{\sigma_\epsilon}\right)^2} - SR_{old}$$

- MKT (0.34), HML (0.20), MOM (0.88), Profitability (0.41), Carry (0.44), ROE (0.80), Investment (0.32)
- average increase of 75% in Sharpe ratios

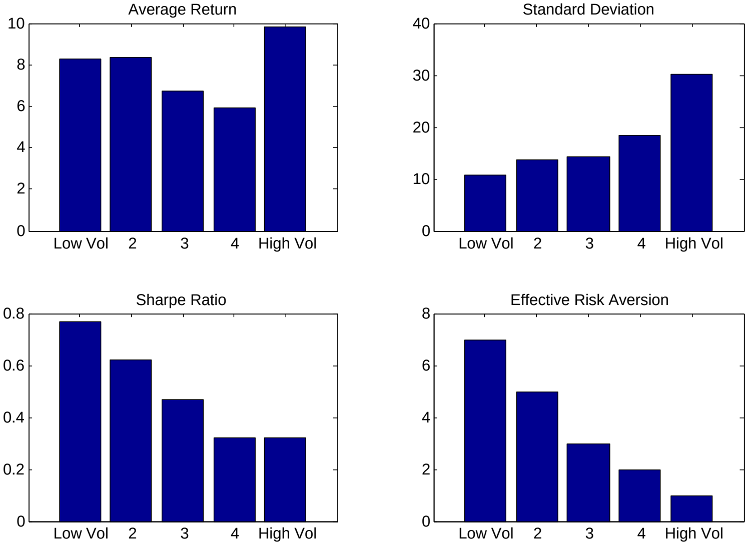
MVE portfolios

1. For given set of factors construct in sample MVE
2. Volatility time the MVE portfolio as before

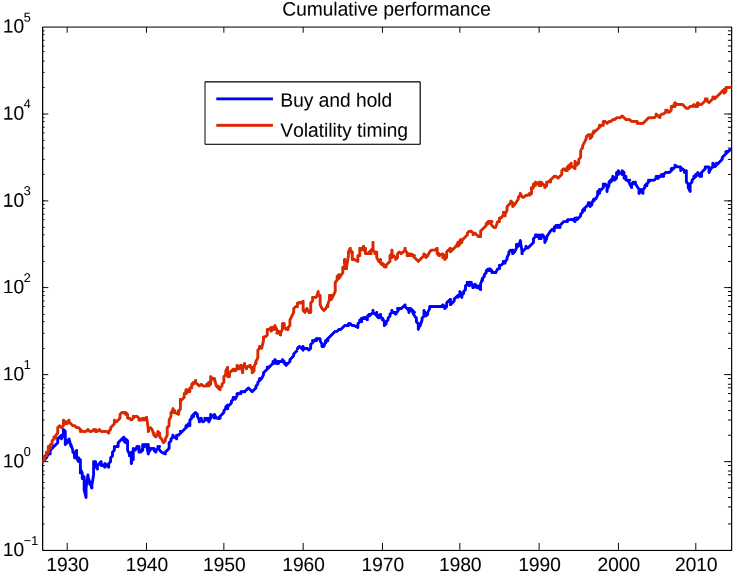
	(1) Mkt	(2) FF3	(3) FF3,Mom	(4) FF5	(5) FF5,Mom	(6) HXZ	(7) HXZ,Mom
Alpha (α)	4.86	4.99	4.04	1.34	2.01	2.32	2.51
	(1.56)	(1.00)	(0.57)	(0.32)	(0.39)	(0.38)	(0.44)
Observations	1,065	1,065	1,060	621	621	575	575
R-squared	0.37	0.22	0.25	0.42	0.40	0.46	0.43
rmse	51.39	34.50	20.27	8.28	9.11	8.80	9.55
Sharpe Ratio	0.42	0.69	1.09	1.20	1.42	1.69	1.73
Appraisal	0.33	0.50	0.69	0.56	0.77	0.91	0.91

$\Rightarrow$ Captures different investment opportunity sets

Strategy works because risk-return trade-off less attractive when vol is high



Cumulative performance for the market return



Robustness of result

1. Look at 20 OECD indices, study many factors in US (Fig 6)
2. Survive transactions costs (Table 5)
3. Other moments besides mean and variance?
   - Generally, 10th and 1st percentiles of vol managed returns are above those for unconditional returns, look at skewness kurtosis also
4. Stronger results with expected vol (Tables 5, 9)
5. Multi-factor regressions: include BAB, risk-parity factors. (Table 10-12)
6. Subsample results (Table 4(b), weaker for 1956-1985)
7. Managed portfolios less exposed to vol *shocks*
8. Works with embedded leverage as well

Subsamples

	(1) Mkt	(2) FF3	(3) FF3 Mom	(4) FF5	(5) FF5 Mom	(6) HXZ	(7) HXZ Mom
α: 1926-1955	8.11	1.94	2.45				
	(3.09)	(0.92)	(0.74)				
α: 1956-1985	2.06	0.99	2.54	0.13	0.71	0.77	1.00
	(2.82)	(1.43)	(1.16)	(0.43)	(0.67)	(0.39)	(0.51)
α: 1986-2015	4.22	5.66	4.98	1.88	2.65	3.03	3.24
	(1.66)	(1.74)	(0.95)	(0.41)	(0.47)	(0.50)	(0.57)

Vol managed portfolios take less risk in recessions

	(1) Mkt^{σ}	(2) HML^{σ}	(3) Mom^{σ}	(4) RMW^{σ}	(5) CMA^{σ}	(6) FX^{σ}	(7) ROE^{σ}	(8) IA^{σ}
MktRF	0.83 (0.08)							
MktRF $\times 1_{rec}$	-0.51 (0.10)							
HML		0.73 (0.06)						
HML $\times 1_{rec}$		-0.43 (0.11)						
Mom			0.74 (0.06)					
Mom $\times 1_{rec}$			-0.53 (0.08)					
RMW				0.63 (0.10)				
RMW $\times 1_{rec}$				-0.08 (0.13)				
CMA					0.77 (0.06)			
CMA $\times 1_{rec}$					-0.41 (0.07)			
Carry						0.73 (0.09)		
Carry $\times 1_{rec}$						-0.26 (0.15)		
ROE							0.74 (0.08)	
ROE $\times 1_{rec}$							-0.42 (0.11)	
IA								0.77 (0.07)
IA $\times 1_{rec}$								-0.39 (0.08)
Observations	1,065	1,065	1,060	621	621	362	575	575
R-squared	0.43	0.37	0.29	0.38	0.49	0.51	0.43	0.49

Leverage constraints

Volatility Timing and Leverage Constraints				
		Alternative volatility managed strategies		
	Buy and hold	With leverage	No leverage (calls)	No leverage (calls + puts)
Sharpe Ratio	0.39	0.59	0.54	0.60
α	–	4.03	5.90	6.67
$s.e.(\alpha)$	–	(1.81)	(3.01)	(2.86)
β	–	0.53	0.59	0.59
Appraisal Ratio	–	0.44	0.39	0.46

Transaction costs

w	Description	$\mid \Delta w \mid$	$E[R]$	α	α After Trading Costs			
					1bps	10bps	14bps	Break Even
$\frac{1}{RV_t^2}$	Realized Variance	0.73	9.47%	4.86%	4.77%	3.98%	3.63%	56bps
$\frac{1}{RV_t}$	Realized Vol	0.38	9.84%	3.85%	3.80%	3.39%	3.21%	84bps
$\frac{1}{E_t[RV_{t+1}^2]}$	Expected Variance	0.37	9.47%	3.30%	3.26%	2.86%	2.68%	74bps
$\frac{1}{max(E[RV_t^2], RV_t^2)}$	RV Above Mean	0.10	9.10%	2.20%	2.19%	2.08%	2.03%	183bps

The dynamics of the risk return tradeoff

Study response to vol shock

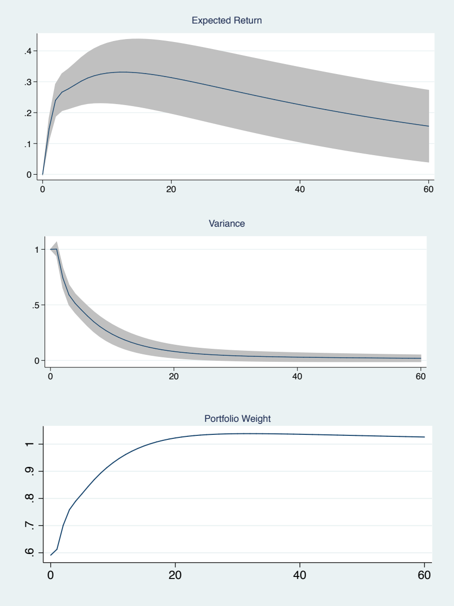
$$w_t = \frac{1}{\gamma}\frac{E_t[R_{t+1}]}{Var_t[R_{t+1}]}$$

Vector Auto Regression with $E_t[R_{t+1}]$ and $Var_t[R_{t+1}]$

- Expected returns formed using CAPE and BaaAaa spread
- Expected variance formed using 3 lags of variance in logs

How much should portfolio weight respond?

Response to 1 std dev variance shock (in months)



Conventional wisdom: don't panic


The New York Times

Stocks and Bonds August 21, 2015 SUBSCRIBE | LOG IN

Advice After Stock Market Drop: Take Some Deep Breaths, and Don't Do a Thing

"If you decide to put a bunch of money in cash...how will you know when to get back in?"

Vanguard (8/15): *What to do during market volatility? Perhaps nothing*

- Vol was at 30-40% in August

Oct/Nov 2008 op-eds Buffett (NYT), Cochrane (WSJ): buying opportunity

- Vol was 60-80%

Our findings

Data: panic driven selling can be beneficial

- NYT: "If you decide to put a bunch of money in cash...how will you know when to get back in?"

  Our answer: get back in when volatility returns to normal (on average 18 months)

- E.g., October 2008: reduce exposure by 90%

- Performance holds up in 1930, 1987, and 2008

Portfolio choice

1. Utility benefits for mean-variance investors

2. Portfolio choice for long-horizon investors

Utility benefits (for a mean-variance investor)

1. Expected return timing (Campbell and Thompson)

$$\Delta U(\%) \approx 35\%$$

   - Instability of predictability an issue (Goyal and Welch)

2. Volatility timing

$$\Delta U(\%) \approx 75\%$$

   - Out of sample vs in sample irrelevant
   - Works for many factors, beyond the aggregate market

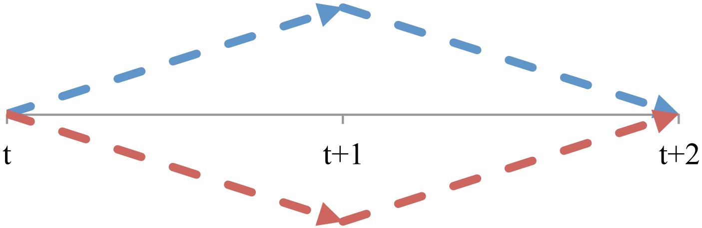
Investor Horizon

Many investors have *long* horizon. Should they time volatility?

1. Long horizon can deviate from myopic/mean variance when returns not iid
2. "Hedging demand" emerges for transitory, mean-reverting shocks ("discount rate" shocks)

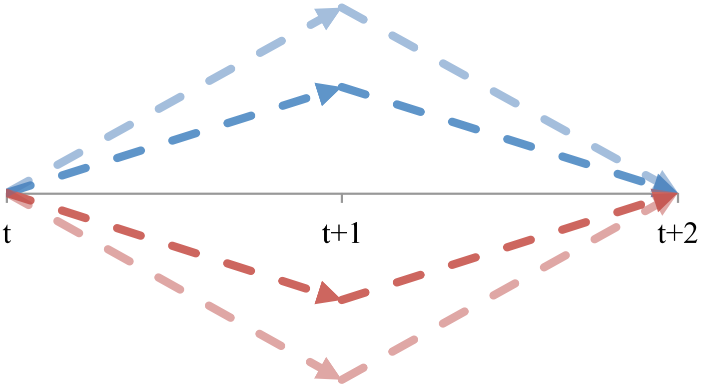
Horizon effects: intuition

Suppose returns were very strongly mean reverting



Horizon effects: intuition

Increase in “discount-rate” vol scarier for short-horizon investor



Horizon effects in practice

Empirically: mean reversion of transitory shocks very *slow*

1. Price dividend ratios highly persistent (Campbell and Shiller)
2. Sharpe ratios increase slowly with horizon (Poterba and Summers)
3. Returns also driven by permanent "cash flow" shocks (Campbell Shiller)

Discount rate and cash flow shocks: convenient terminology to describe the behavior of returns

Optimal behavior of long horizon investor

1. CRRA investor, vary horizon
   - Paper: generalize to Epstein Zin
2. Calibrate process for returns, expected returns, volatility, etc
3. Consider cases where variation in volatility is driven by:
   i discount rate vol,
   ii cash flow vol,
   iii constant mix of both

Portfolio choice: optimal portfolio

$$portfolio = Myopic + hedging\ demand$$

$$w_t^* = \frac{1}{\gamma}\frac{\mu_t - r}{\sigma_{R,t}^2} + \frac{V_x \kappa_x}{\gamma}\frac{\sigma_t^2(discount\ rate\ shocks)}{\sigma_{R,t}^2}$$

Portfolio choice: optimal portfolio

$$\begin{aligned} \textit{portfolio} &= \textit{Myopic} + \textit{hedging demand} \\ w_t^* &= \frac{1}{\gamma}\frac{\mu_t - r}{\sigma_{R,t}^2} + \frac{V_x \kappa_x}{\gamma}\frac{\sigma_t^2(\textit{discount rate shocks})}{\sigma_{R,t}^2} \end{aligned}$$

Numerical result: optimal portfolio well approximated by

buy-and-hold + volatility-managed portfolio

$$E[w_t^* | \sigma_t^2] \approx a + b\frac{\mu_x}{\gamma} \times \frac{1}{\sigma_{R,t}^2}$$

Portfolio choice: optimal portfolio

$$portfolio = Myopic + hedging\ demand$$

$$w_t^* = \frac{1}{\gamma}\frac{\mu_t - r}{\sigma_{R,t}^2} + \frac{V_x \kappa_x}{\gamma}\frac{\sigma_t^2(discount\ rate\ shocks)}{\sigma_{R,t}^2}$$

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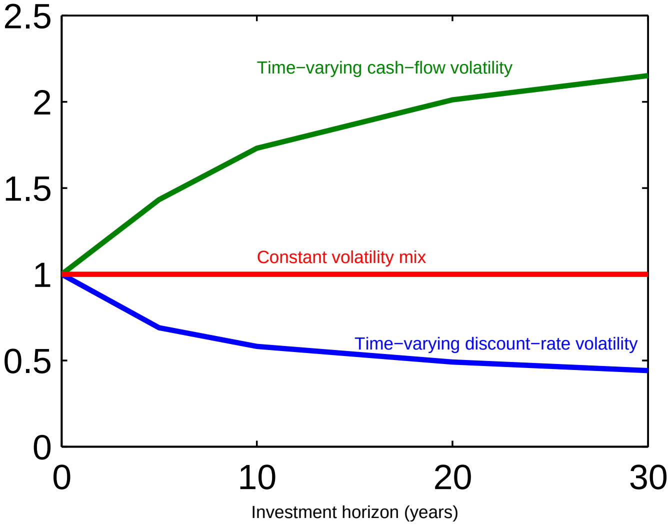
$$E[w_t^*|\sigma_t^2] \approx a + b\frac{\mu_x}{\gamma} \times \frac{1}{\sigma_{R,t}^2}$$

- Weights, a and b, describe optimal portfolio
- extent of volatility timing described by b
- Mean-variance or log: b=1 and a=0.
- Long horizon: b depends on driver of volatility

Optimal policy b by horizon for CRRA w/ $\gamma = 10$



General equilibrium models

$$E_t[R_{t+1}] \approx \gamma_t \sigma_t^2$$

1. **Models:** $cov(\gamma_t, \sigma_t^2) \geq 0$

   (habits, intermediary, prospect theory, long run risk, rare disasters)

2. **Data:** $cov(\gamma_t, \sigma_t^2) < 0$ for all factors and MVE combinations of factors

- Risk aversion / price of risk low in high vol states?
- Paper: show how to estimate $cov(\gamma_t, \sigma_t^2)$, show high $\alpha \Rightarrow cov < 0$

Covariance and alpha

1. Define factor price of risk

$$\gamma_t = \frac{E_t[R_{t+1}]}{Var_t(R_{t+1})} \qquad \gamma = \frac{E[R_{t+1}]}{Var(R_{t+1})}$$

2. It follows that

$$cov(\gamma_t, \sigma_t^2) = -\left(\frac{\alpha - E[\mu_t](E[\beta_t] - \beta)}{E[\beta_t]} + \frac{E[\gamma_t]}{E[1/\sigma_t^2]}\left(E[\sigma_t^2]E[1/\sigma_t^2] - 1\right)\right)$$

   more negative cov. between variance and price of risk, larger alpha

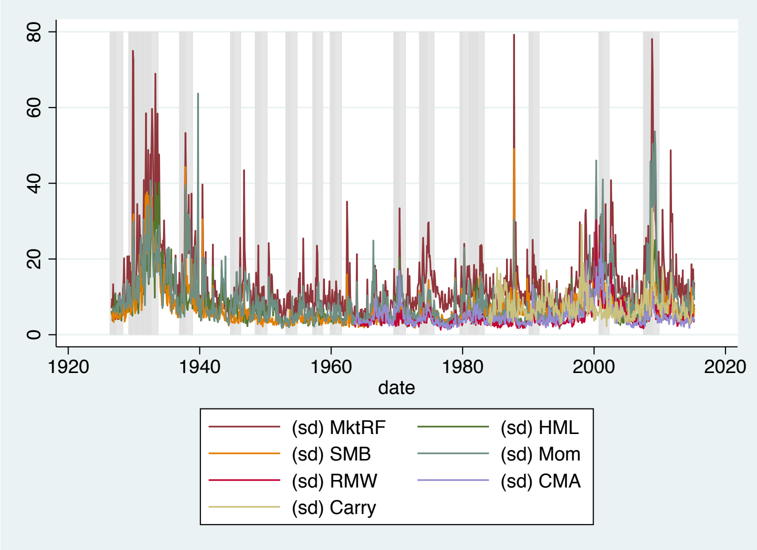
3. Regression of future returns on variance recovers $cov(\gamma_t, \sigma_t^2)$

Risk aversion (price of risk) negatively correlated with vol

	$cov(\gamma_t, \sigma_t^2)$		
	Mean	Std Err	95% CI
MktRF	-0.50	0.17	[-0.84, -0.17]
SMB	0.03	0.12	[-0.21, 0.28]
HML	-0.27	0.13	[-0.52, -0.01]
Mom	-1.51	0.19	[-1.89, -1.12]
MVE	-0.62	0.09	[-0.80, -0.45]
RMW	-0.20	0.08	[-0.36, -0.05]
CMA	-0.06	0.07	[-0.19, 0.07]
MVE2	-0.29	0.06	[-0.41, -0.17]
Carry	-0.20	0.07	[-0.33, -0.06]

also holds at the quarterly / annual frequency

Vol strongly countercyclical for all factors



Literature

- Barroso and Santa-Clara (2015) on momentum timing
- Fleming, et al (2001) on volatility timing
  - Daily frequency, cross-sectional, estimate full covariance matrix and assumption about expected returns
- Low risk anomalies in the cross-section: Frazzini and Pedersen (2014), Ang, et. al. (2006)
  - Our portfolios managed in time-series only. Exploit lack of risk-return tradeoff *over time*
- Portfolio choice
  - Viceira and Chacko 2005: Stochastic cash flow vol, expected returns constant in their model
  - Viceira and Campbell 1999, Barberis 2001, etc.: Constant vol, time-varying expected returns

Conclusion

Risky asset demand: $w_t = E_t[R_{t+1}]/\sigma_t^2$

1. Vol managed portfolios across many factors
   - large α's
   - Sharpe ratios increase $\approx$ 75%
   - Take less risk in recessions and after market crashes
2. Portfolio choice: reduce exposure when volatility is high
   - Increase utility $\approx$ 75%
   - Long horizon still vol time (less for transitory vs permanent shocks)
3. General equilibrium puzzle: price of risk low when vol is high

Portfolio choice: stochastic environment

$$\begin{aligned}
\text{Returns: } dR_t &= (r + x_t)dt + \sqrt{y_t} D_R dB_t + F_R dZ_t && (1)\\
\text{Expected returs: } dx_t &= \kappa_x(\mu_x - x_t)dt + \sqrt{y_t} D_x dB_t + F_x dZ_t && (2)\\
\text{Variance: } dy_t &= \kappa_y(\mu_y - y_t)dt + \sqrt{y_t} D_y dB_t && (3)
\end{aligned}$$

- dB_t and dZ_t are 3 by 1 independent Brownian motions
- D_R, D_x, F_R, Fx consistent with no long run effect of dx_t shock
- Calibrate model to match the aggregate market factor

  (1) one year return R^2, (2) persistence of expected return shocks, (3) volatility, (4) volatility of volatility, (5) persistence of volatility, (6) co-variance between volatility and future returns, (7) co-variance between volatility shocks and realized returns, and (8) unconditional Sharpe ratio.

Portfolio choice: preferences

Duffie-Epstein (Epstein Zin) utility

$$
\begin{aligned}
J_t &= E_t\left[\int_t^{\infty} f(C_s, J_s)ds\right] \\
f(C,J) &= \rho\frac{1-\gamma}{1-\frac{1}{\psi}}J \times \left[\left(\frac{C}{((1-\gamma)J)^{\frac{1}{1-\gamma}}}\right)^{1-\frac{1}{\psi}} - 1\right]
\end{aligned}
$$

- Value function has the usual form: $J(W,x,y) = \frac{W^{1-\gamma}}{1-\gamma}e^{V(x,y)}$

Portfolio choice: preferences

1. Use ρ to capture variation in investment horizon
   - Perpetual youth model (Blanchard, Garleanu and Panageas)
   - ρ consistent with half-life ranging from 5 to 30 years.
2. Risk-aversion: $\gamma \in \{5, 10\}$
3. IES: $\psi \in \{1/\gamma, 0.5, 1, 1.5\}$

Portfolio choice: optimal portfolio

Volatility mix view:

portfolio=Myopic + expect. return hedging dem.+ vol. hedging dem.

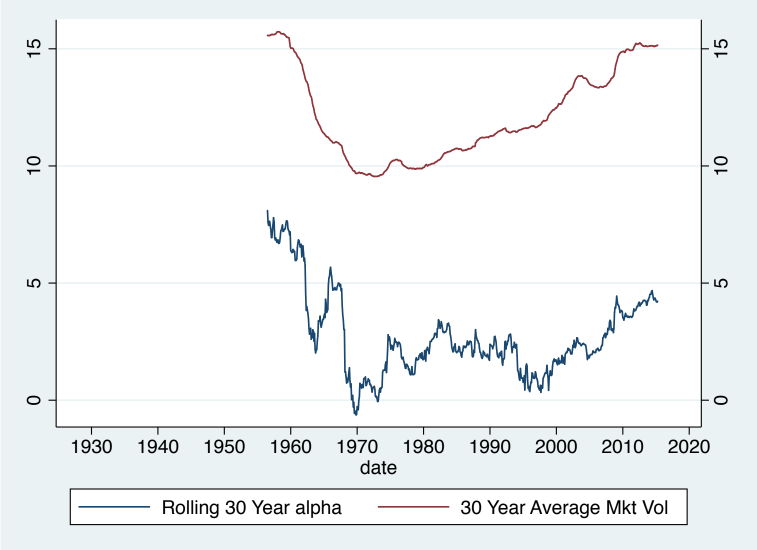
$$w_t^* = \frac{1}{\gamma}\frac{E_t[dR_t] - r}{Var_t(dR_t)} + \frac{V_x \kappa_x}{\gamma} DR_{share}(y_t) + \frac{V_y}{\gamma}\beta_{dy_t, dR_t}$$

$DR_{share}(y_t)$: share of return vol driven by discount rate shocks

$$DR_{share}(y_t) = \frac{D_R' D_x \times y_t F_R' F_x}{\kappa_x (D_R' D_R \times y_t F_R' F_R)}$$

- $\frac{\partial DR_{share}(y)}{\partial y} > 0$: volatility driven by discount-rate volatility
- $\frac{\partial DR_{share}(y)}{\partial y} < 0$: volatility driven by cash-flow volatility

Rolling Alphas



Does strategy load on the variance risk premium?

- Is strategy highly exposed to a large “surprise” in volatility?
- For Mkt: One-standard deviation increase in variance leads to a 1.4% drop in the buy-and-hold portfolio and only 0.7% drop in the volatility managed counterpart
- Similar for other factors $\rightarrow$ vol of vol high when vol also high

Betting against beta

	(1) Mkt$^{\sigma}$	(2) SMB$^{\sigma}$	(3) HML$^{\sigma}$	(4) Mom$^{\sigma}$	(5) RMW$^{\sigma}$	(6) CMA$^{\sigma}$	(7) MVE$^{\sigma}$
MktRF	0.60						
	(0.05)						
BAB	0.09	0.01	0.02	-0.07	-0.13	-0.06	0.04
	(0.06)	(0.05)	(0.05)	(0.04)	(0.02)	(0.02)	(0.02)
SMB		0.61					
		(0.09)					
HML			0.56				
			(0.07)				
Mom				0.47			
				(0.06)			
RMW					0.65		
					(0.08)		
CMA						0.69	
						(0.04)	
MVE							0.57
							(0.04)
Constant	3.83	-0.77	2.05	13.52	3.97	0.94	4.10
	(1.80)	(1.10)	(1.15)	(1.86)	(0.89)	(0.71)	(0.85)
Observations	996	996	996	996	584	584	996
R-squared	0.37	0.37	0.31	0.21	0.40	0.46	0.33
rmse	52.03	31.36	35.92	51.73	19.95	17.69	26.01

Risk Parity

Follow Ansess Frazzini Pedersen, construct

$$RP_{t+1} = b_t' f_{t+1} \tag{4}$$

Where

$$b_{i,t}' = \frac{1/\sigma_{i,t}}{\sum 1/\sigma_{i,t}} \tag{5}$$

Control for this factor in our regressions, alphas change very little

Our approach keeps relative weights constant:

$$b_{i,t}' = \frac{b_i}{var_t(b' f_{t+1})} \tag{6}$$

Risk-parity factor

	(1) Mkt	(2) FF3	(3) FF3 Mom	(4) FF5	(5) FF5 Mom	(6) HXZ	(7) HXZ Mom	(8) BAB^{σ}
Alpha (α)	4.86	5.00	4.09	1.32	1.97	2.03	2.38	5.67
	(1.56)	(1.00)	(0.57)	(0.31)	(0.40)	(0.32)	(0.44)	(0.98)
Observations	1,065	1,065	1,060	621	621	575	575	996
R-squared	0.37	0.23	0.26	0.42	0.40	0.50	0.44	0.33
rmse	51.39	34.30	20.25	8.279	9.108	8.497	9.455	29.73