Volatility Expectations and Returns

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Overview

Risk-return tradeoff a cornerstone of asset pricing

$$E_t[r_{t+1}^{mkt}] - r_{f,t} = \gamma_0 + \gamma\sigma_t^2$$

- Empirically, risk-return tradeoff is weak / wrong sign
- ...but strong in our models (e.g, power utility, habits, long run risk, intermediation, disasters...)

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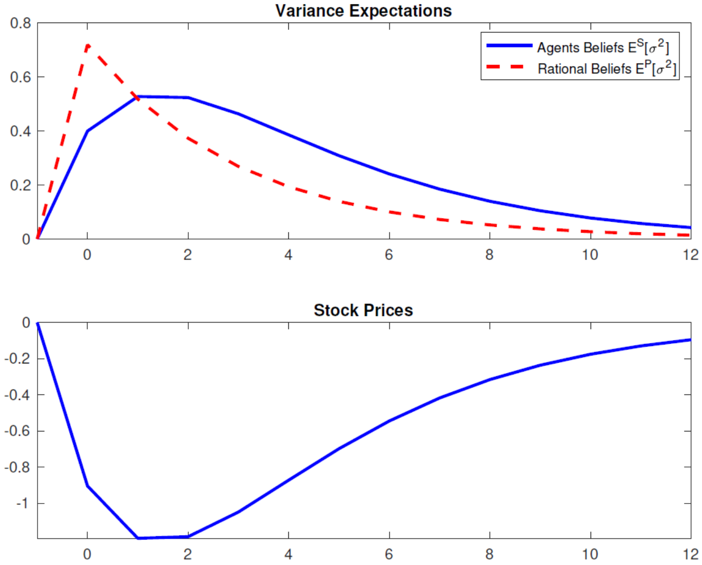
This paper: Allow for biased beliefs about risk

- Survey evidence: agents place too much weight on past volatility, not enough on current volatility
- Initial underreaction to volatility and delayed overreaction
- Pattern shows up in volatility claims (e.g., variance swap / VIX futures)
- Put beliefs into GE model, can explain weak risk-return tradeoff + many other patterns

Intuition of the Paper: Response to Variance Shock



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Mainstream models match (2), extensions can match (3) (e.g., Bollerslev Tauchen Zhou 2009), but strongly counter-factual for (1), (4), and (5)

Outline

1. Survey evidence that (some) agents have slow-moving expectations about stock market return variance / risk
   - Show survey beliefs not fully rational

2. Stylized facts (old and new) on relation between stock returns and variance
   - Current mainstream asset pricing models (e.g., long-run risk, habit, intermediary, disaster) don't match the patterns

3. Embed beliefs in equilibrium asset pricing model
   - Match puzzling facts about return volatility, the VIX

- Show dynamics of variance premiums in the cross-section are consistent with models' belief channel

Survey Data

Two surveys:

1. CFO survey of Graham and Harvey (quarterly since 2001)
   - Respondents asked for 90-10th percentile distribution for next year's stock returns
   - We square this difference as a measure of variance

2. Shiller survey (monthly since 2001)
   - Respondents asked for probability of stock market crash over the next 6 months, such as that seen in 1987

Neither measure perfect, we view these as correlated with return variance expectations

Relevance: beliefs *do* translate to portfolio decisions (Giglio et al, 2019)

Estimating Beliefs

Estimate ϕ in CFO survey:

$$Survey_t^{(1yr)} = a + b \sum_{j=0}^{J} \phi^j rv_{t-j} + \varepsilon_{t,t+12}$$

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Panel A: Dependence on Past Variance (ϕ)		
Source	Survey	Future Variance
CFO	0.87***	-0.16
	(0.11)	(0.38)
N	69	69
R^2	0.74	0.36
$t(\phi_{survey} - \phi_{fv})$		(2.58)

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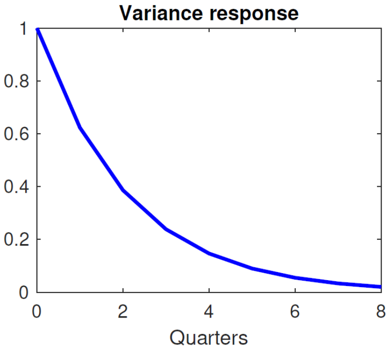
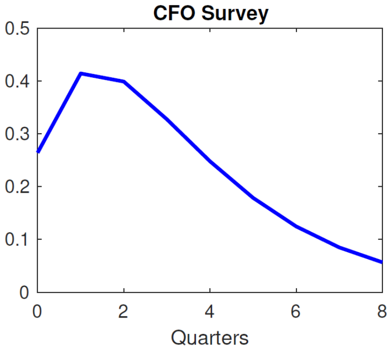
Similar findings using VIX^2 (mkt expectation of variance), recover $\phi \approx 0.4$

Estimating Beliefs: VAR evidence

Bi-variate VAR(1): realized variance and CFO survey

- Impulse response from variance shock





Do "mistakes" show up in prices?

Monthly data, 1990-2020. Closely related to Cheng (2019)

- Look at returns that are short volatility or variance
- Idea: when investors believe vol higher than rational forecast, returns to shorting vol should be high
- $\hat{\sigma}^2_{t-1}$: real-time variance forecast from intraday data

	(1) Vix Fut	(2) Straddle	(3) Var Swap	(4) Var Swap (TJ)	(5) $VIX^2_{t-1} - \sigma^2_t$	(6) σ^2_t	(7) VIX^2_{t-1}
$\hat{\sigma}^2_{t-1}$	-5.04*****	-29.18**	-40.62***	-55.90***	-1.26***	1.53***	0.30**
	(1.78)	(11.49)	(14.16)	(14.52)	(0.49)	(0.54)	(0.12)
$\Sigma_{j=1}^{6} \phi^j \hat{\sigma}^2_{t-j}$	5.11**	36.57**	42.90**	58.96***	1.23***	-0.58	0.64***
	(2.13)	(14.24)	(20.19)	(16.86)	(0.46)	(0.46)	(0.07)
N	194	292	264	282	334	334	335
R^2	0.0523	0.0240	0.00682	0.00496	0.172	0.469	0.790

Market risk-return trade-off

Monthly data, 1990-2020. Updating existing evidence.

- Excess market returns vs variance
- VRP strongly predicts returns, VIX and conditional variance does not

	(1) Market	(2) Market	(3) Market	(4) Market
$\hat{\sigma}^2_{t-1}$	-0.59		-2.21	-3.44***
	(0.80)		(1.38)	(0.87)
VIX^2_{t-1}		0.41		3.69***
		(1.16)		(1.25)
$\Sigma^6_{j=1}\phi^j\hat{\sigma}^2_{t-j}$			2.17	
			(2.11)	
N	340	363	335	340
R^2	2.04e-05	-0.00170	0.00105	0.0211

Robustness of Facts

- Volatility (instead of variance) or log of variance, largely same takeaways, slightly weaker. Robust: VRP predicts returns. RV (if anything) negatively predicts (significance weaker in other specifications), VIX alone is a weak return predictor.
  - Suggests high variance periods are important
- Market returns on variance results are robust to the post-WW2 sample, less so for Great Depression (coefficient still negative, insignificant)
- Robust internationally, 9 developed countries. Weaker evidence in Japan.

The Model: Cash flows

Aggregate log divided growth:

$$\begin{aligned} \Delta d_t &= \mu + \sigma_t \varepsilon_t, \\ \sigma_t^2 &= \bar{v} + \rho\left(\sigma_{t-1}^2 - \bar{v}\right) + \omega \eta_t. \end{aligned}$$

where ε, η uncorrelated standard Normals

- Variance negative? Done to be consistent with prior lit, can use Gamma shocks to avoid and main results unaffected
- Note:

$$\begin{aligned} E_{t-1}\left[\sigma_t \varepsilon_t\right] &= 0, \\ Var_{t-1}\left(\sigma_t \varepsilon_t\right) &= E_{t-1}\left[\sigma_t^2\right] = \bar{v} + \rho\left(\sigma_{t-1}^2 - \bar{v}\right) \end{aligned}$$

Beliefs

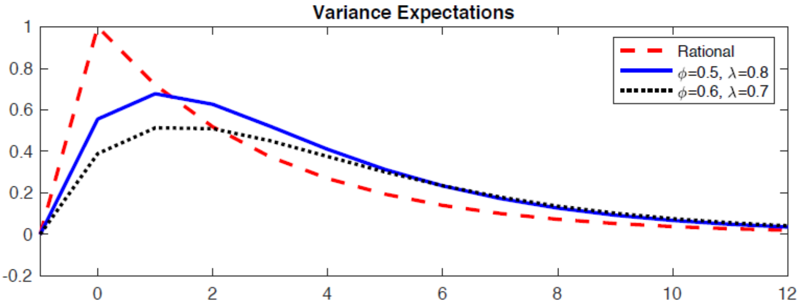
Agents observe σ_t^2 and ε_t at time t.

Beliefs about future variance:

$$
\begin{aligned}
E_{t-1}^{S}\left[\sigma_t^2\right] &= \bar{v} + \lambda x_{t-1}, \\
x_t &= \phi x_{t-1} + (1-\phi)\left(\sigma_t^2 - \bar{v}\right) \\
&= (1-\phi)\sum_{j=0}^{\infty} \phi^j \left(\sigma_{t-j}^2 - \bar{v}\right)
\end{aligned}
$$

- Rational: $\phi = 0$, $\lambda = \rho$
- ϕ : degree of extrapolation, weights on past lags
- λ : scale, affects subjective autocorrelation $(\phi + (1-\phi)\lambda)$, consistent with 1- and 10-year survey data

Dynamics of Variance Expectations



- Extrapolation: E.g., Malmendier and Nagel (2011, 2015), Greenwood and Shleifer (2014), Glaeser and Nathanson (2017).
- Under/overreaction: E.g., Daniel, Hirshleifer, and Subrahmanyam (1998), Barberis, Shleifer, and Vishny (1998), Landier, Ma, and Thesmar (2019), Brooks, Katz, and Lustig (2019), Bordalo et al (2020)
- Eqm models: E.g., Barberis, Greenwood, Jin, and Shleifer (2015), Collin-Dufresne, Johannes, and Lochstoer (2017), Nagel and Xu (2019)

Equilibrium

Assume a representative stock-holder with Epstein-Zin preferences

- Agent's consumption = aggregate dividends
- Reminiscent of intermediary, limited-participation based model, simplified exposition

Law of one price holds under agents measure

- Subjective SDF:

$$M_t = \beta^\theta e^{-\frac{\theta}{\psi}\Delta d_t + (\theta - 1) r_t}$$

  where $\theta = (1-\gamma) / (1 - 1/\psi)$
- Then:

$$E^S_{t-1}\left[M_t R_t\right] = 1 \text{ for all } R \text{ and } t$$

Use Campbell-Shiller approximation ($r_t \approx \kappa_0 + \kappa pd_t - pd_{t-1} + \Delta d_t$) to get exponential affine model (as in, e.g., Bollerslev, Tauchen, and Zhou (2009))

Equilibrium Quantities

- Log P/D ratio:

$$pd_t = c - Ax_t$$

where $A = -\frac{1}{2}\frac{\lambda(1-\gamma)(1-1/\psi)}{1-\kappa(\phi+(1-\phi)\lambda)}$. $A > 0$, price down when vol up

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- Subjective risk premium:

$$E^S_{t-1}\left[r_t - r_{f,t}\right] = \left(\gamma - \frac{1}{2}\right) E^S_{t-1}\left[\sigma^2_t\right] + \delta_r$$

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- Objective risk premium:

$$E^P_{t-1}\left[r_t - r_{f,t}\right] = E^S_{t-1}\left[r_t - r_{f,t}\right] - \kappa\left(1-\phi\right) A\left(E^P_{t-1}\left[\sigma^2_t\right] - E^S_{t-1}\left[\sigma^2_t\right]\right)$$

Variance Risk Premium Dynamics

- Price the variance claim:

$$RV_t = \Theta + \sigma_t^2$$

- Implied variance:

$$IV_{t-1} = E_{t-1}^{S}\left[R_{f,t} M_t RV_t\right]$$

- Ex ante objective VRP:

$$\begin{aligned} VRP_{t-1} &= IV_{t-1} - E_{t-1}^{P}\left[RV_t\right] \\ &= \delta_{IV} + E_{t-1}^{S}\left[\sigma_t^2\right] - E_{t-1}^{P}\left[\sigma_t^2\right] \end{aligned}$$

- Thus, error in expectations shows up in both objective equity and variance risk premiums

Model Calibration

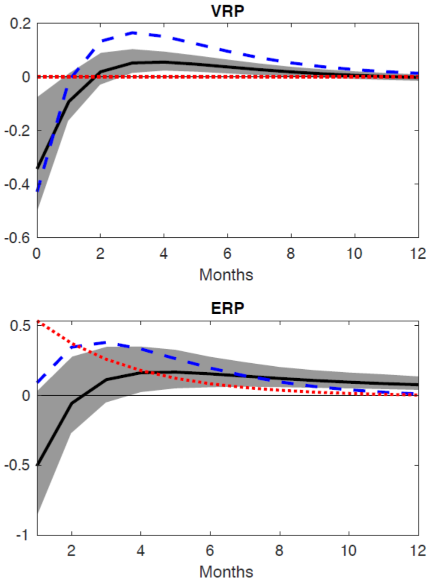
Panel A: Parameters

Parameter	Description	Value	Targeted Moment(s)
γ	Risk Aversion	3	Equity Premium
ψ	Elasticity of Intertemporal Substitution	2.2	Literature / VRP
$\bar{v}$	Unconditional Variance (Monthly)	0.25%	Data
ρ	Persistence of Variance	0.71	Data
ω	Volatility of Variance Shocks (Monthly)	0.31%	Data
ϕ	Expectation Stickiness	0.5	VIX / Surveys
λ	Scale of Expectations	0.8	Vol of VIX / Surveys

Panel B: Moments

Moment	Description	Model	Data
$E[r_m] - r_f$	Equity Premium (Annual)	7.9%	7.7%
$\sqrt{E[RV_t]}$	Square Root Avg. Variance (Annual)	18%	18%
$\rho(RV_t, RV_{t-1})$	Persistence of Variance (Monthly)	0.71	0.71
$\sigma(RV_t)$	Volatility of Variance (Monthly)	0.44%	0.44%
$\sigma(VIX_t^2)$	Volatility of VIX^2 (Monthly)	0.33%	0.35%
$\rho(VIX_t^2, RV_t)$	Correlation RV and VIX^2 (Monthly)	0.88	0.85
$\rho(VIX_t^2, VIX_{t-1}^2)$	Persistence of VIX^2 (Monthly)	0.91	0.84
$\alpha\left(\frac{c}{RV_{t-1}} r_{m,t}, r_{m,t}\right)$	Volatility-Managed Alpha (Moreira Muir)	1.4%	4.9%
$\rho(r_{m,t}, \widetilde{RV}_t)$	Correlation of Returns and Vol Shocks	-0.20	-0.38

Model Dynamics vs. Data



Comparing to Bollerslev, Tauchen, and Zhou

In BTZ, variance dynamics are:

$$
\begin{aligned}
\sigma_t^2 &= \bar{v} + \rho\left(\sigma_{t-1}^2 - \bar{v}\right) + \sqrt{q_{t-1}}\eta_t \\
q_t &= \varphi_1 + \varphi_2 q_{t-1} + \varphi_3 \sqrt{q_{t-1}}\delta_t
\end{aligned}
$$

BTZ agents are rational, EZ preferences with $\gamma > \psi^{-1}$, $\psi > 1$

(Note: BTZ state in their paper that their model has a counter-factually strong risk-return trade-off and leave this issue for future research. We compare to them to show how we differ from this benchmark model.)

Comparison of Dynamics: LM vs. BTZ

	Data	Model			
		BTZ	$\phi = 0.3$	$\phi = 0.5$	$\phi = 0.8$
	(1)	(2)	(3)	(4)	(5)
Risk Return Tradeoff					
$\hat{\sigma}^2_{t-1}$	-2.21	10	-0.23	-1.52	-1.05
	(1.38)				
$\Sigma_{j=1}^{6} \phi^j \hat{\sigma}^2_{t-j}$	2.17	0	3.02	3.74	1.78
	(1.42)				
R^2	0.1%	7.9%	3.5%	3.4%	1.4%
Forecasting Returns with Variance Risk Premium					
VRP_{t-1}	3.47	3.94	7.40	2.58	1.42
	(0.92)				
R^2	2.3%	1.2%	1.9%	1%	0.5%
Forecasting Returns with E[RV] and VIX					
$\hat{\sigma}^2_{t-1}$	-3.44	6.06	-2.28	-2.21	-2.02
	(0.87)				
VIX^2_{t-1}	3.69	3.94	4.78	4.71	4.52
	(1.25)				
R^2	2.1%	8%	4%	3.6%	3.4%
Expected Variance Risk Premium ($VRP_{t-1} = VIX^2_{t-1} - E_{t-1}[\sigma^2_t]$)					
$\hat{\sigma}^2_{t-1}$	-1.26	0	-0.25	-0.70	-0.92
	(0.49)				
$\Sigma_{j=1}^{6} \phi^j \hat{\sigma}^2_{t-j}$	1.23	0	0.39	0.80	0.58
	(0.46)				
Correlation: VIX^2 and Realized Variance					
	0.86	0.99	0.96	0.89	0.73

Volatility-Managed Portfolios

Moreira and Muir (2017) show that one can improve factor Sharpe ratios by volatility timing

- For market

$$R_{t+1}^{timed} = \frac{c}{RV_t} R_{mkt,t+1}^{excess}.$$

Alpha can be approximated as:

$$\alpha = -\frac{c}{E\left[RV_t\right]} Cov\left(E_t\left[RV_{t+1}\right], \frac{\mu_t}{E_t\left[RV_{t+1}\right]}\right)$$

where $\mu_t \equiv E_t\left[R_{t+1}^{excess}\right]$

- No timing benefit if $\mu_t = const \times E_t\left[RV_{t+1}\right]$
- Good timing benefit if $\mu_t \downarrow$ when $E_t\left[RV_{t+1}\right] \uparrow$, the case in our model
- Moreira Muir 2017: habits, LRR, disaster, intermediary have $\alpha \leq 0$

Longer-maturity variance claims

Long-maturity variance claims respond in the same manner

- Consistent with survey evidence of extrapolation
- Regressing 10-yr on 1-yr survey: monthly implied autocorrelation ≈0.96

	Panel A: Model Coefficients by Maturity				
	(1)	(2)	(3)	(6)	(12)
σ^2_{t-1}	-0.34	-0.20	-0.15	-0.14	-0.12
$\Sigma_{j=1}^{6}\phi^j\hat{\sigma}^2_{t-j}$	0.54	0.31	0.24	0.17	0.14
Adj. R^2	15.9%	15.9%	15.9%	15.9%	15.9%
	Panel B: Data Variance Swaps				
	(1)	(2)	(3)	(6)	(12)
σ^2_{t-1}	-0.90***	-0.78***	-0.66***	-0.45***	-0.22***
	(0.24)	(0.19)	(0.15)	(0.09)	(0.06)
$\Sigma_{j=1}^{6}\phi^j\hat{\sigma}^2_{t-j}$	1.02**	0.85**	0.69***	0.40**	0.15
	(0.47)	(0.36)	(0.26)	(0.17)	(0.12)
N	264	264	264	264	264
Adj. R^2	0.7%	1.0%	1.4%	1.3%	0.0%
	Panel C: Data Straddle Returns				
	(1)	(2)	(3)	(6)	(12)
σ^2_{t-1}	-0.40***	-0.41***	-0.27***	-0.17***	-0.16***
	(0.12)	(0.07)	(0.07)	(0.05)	(0.03)
$\Sigma_{j=1}^{6}\phi^j\hat{\sigma}^2_{t-j}$	0.58***	0.47***	0.32***	0.22***	0.15***
	(0.21)	(0.10)	(0.09)	(0.06)	(0.04)
N	264	264	264	264	264
Adj. R^2	1.1%	2.9%	1.7%	1.4%	1.7%

Cross-section of Options and Variance

If agents misestimate the variance dynamics, one would expect this to also show up in the cross-section of options

- Also make mistake with regards to idiosyncratic volatility
- OptionMetrics data from 1996-2017
- Get ATM implied vol from end of month options, realized variance is from daily stock returns

	ERP	Vol	VRP	ERP	Vol	VRP
$\Delta_6\sigma_t^2$	-0.129	0.253^{***}	-0.104^{***}	-0.040	0.188^{***}	-0.070^{***}
	(0.137)	(0.067)	(0.036)	(0.075)	(0.046)	(0.022)
N	536,726	536,726	536,726	536,726	536,726	536,726
Adj R^2	0.001	0.010	0.002	0.159	0.060	0.024
Time FE	N	N	N	Y	Y	Y

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- Heterogeneous agent models can have declining variance in very bad states. But, a calibrated model must also match the unconditionally negative correlation between returns and variance shocks so hard to make former effect strong enough. Also not clear how one would get the delayed, hump-shaped response, or the cross-sectional evidence.

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- Nagel et al (2017) show investors do react to volatility, and more sophisticated agents react more. Arguably consistent with our model (though we only have one agent)
- Survey evidence has to be dismissed for any rational model, counter-intuitive risk premium dynamics

Conclusion

Slow-moving beliefs about variance can match many puzzling empirical facts surrounding volatility and risk premiums

- Results in underreaction, delayed overreaction to volatility

Propose model where agents extrapolate from past volatility, estimate the degree to which they do so in survey data

- Match weak risk-return trade-off, the dynamic responses of the equity and variance risk premiums following shocks to variance
- Account for the fact that shocks to volatility are associated with negative realized returns through a discount rate channel
- The variance risk premium predicts returns much more strongly than either variance or implied variance, as in the data

Survey evidence directly supports the idea that agents have slow moving expectations about volatility, as does evidence at the firm level